Physics ∩ ML, online, 17 June 2020

Deep Learning and Quantum Gravity

Koji Hashimoto (Osaka U.)

"Neural ODE and Holographic QCD"
ArXiv:2006.00712 w/ H.Y.Hu, Y.Z.You (UCSD)
"Deep Learning and AdS/QCD"
ArXiv:2005.02636 w/ T. Akutagawa, T. Sumimoto (Osaka u)
"Deep Boltzmann Machine and AdS/CFT"
ArXiv:1903.04951
"Deep Learning and AdS/CFT" "Deep Learning and Holographic QCD"
ArXiv:1802.08313, 1809.10536
w/ S. Sugishita (Kentucky), A. Tanaka, A. Tomiya (RIKEN)

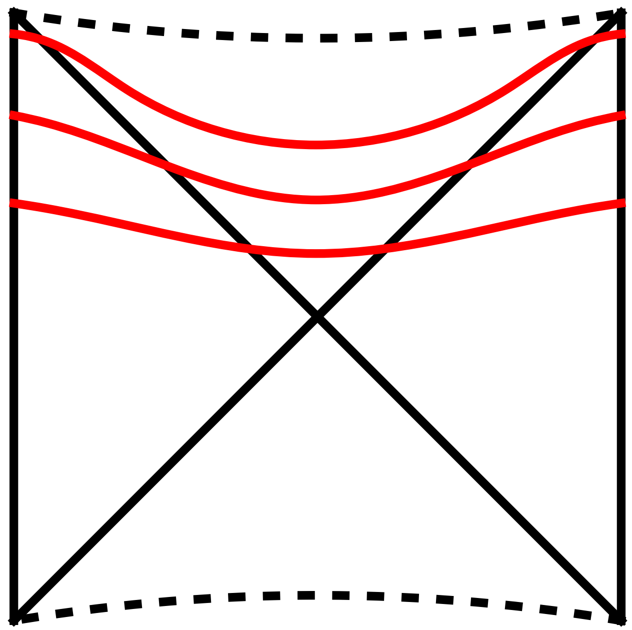
> Osaka CTSR – RIKEN iTHES/iTHEMS - Kavli IPMU
> Joint symposium

Deep learning and physics

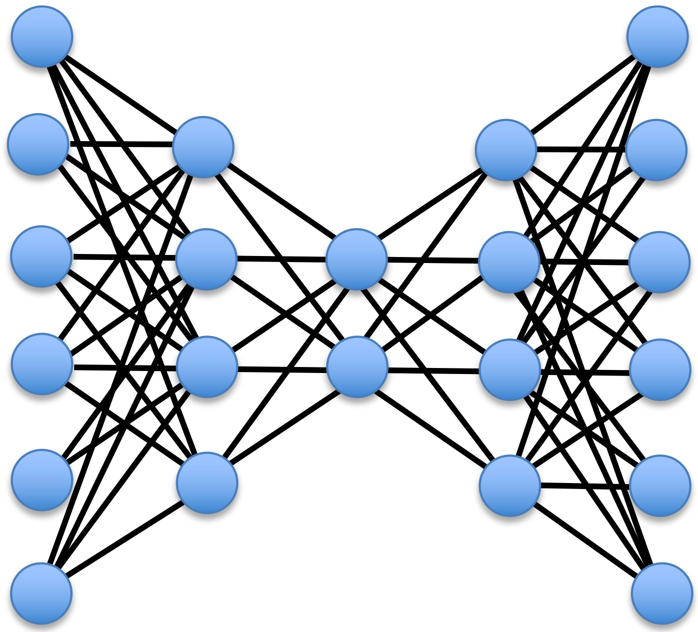
> Venue: Nambu hall, Osaka university
> Date: June 5 (Mon), 2017, 13:00-18:00
> Invited speakers :
> S. Amari (RIKEN)
> S. Ikeda (ISM / Kavli IPMU)
> Y. Kawahara (Osaka U. / RIKEN)
> M. Taki (RIKEN)
> A. Tanaka (RIKEN)
> T. Ohtsuki (Sophia U.)
> N. Suzuki (Kavli IPMU)

> Organizers:
> K. Hashimoto (Osaka U.)
> T. Hatsuda (RIKEN iTHES/iTHEMS)
> H. Murayama (Kavli IPMU)

Similarity!?



Wormholes in Penrose diagram
of maximally extended eternal
AdS Schwarzschild black hole
[Iizuka, Sugishita, KH `17]

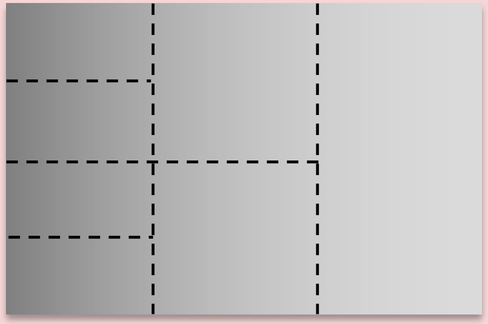


Deep Autoencoder

Roadmap

General spacetime

Anti de Sitter spacetime



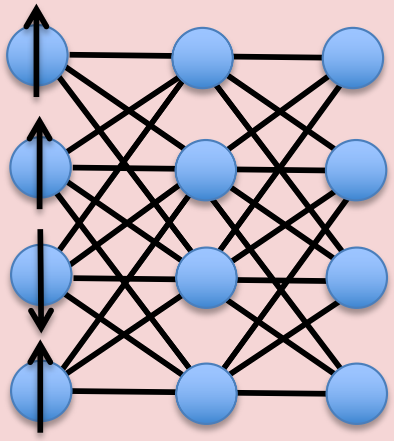
Quantum gravity in (d+1)-dim.

‘tHooft `93
Susskind `94
Maldacena `97

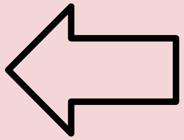
||

Quantum mechanics in d-dim.

|| ?

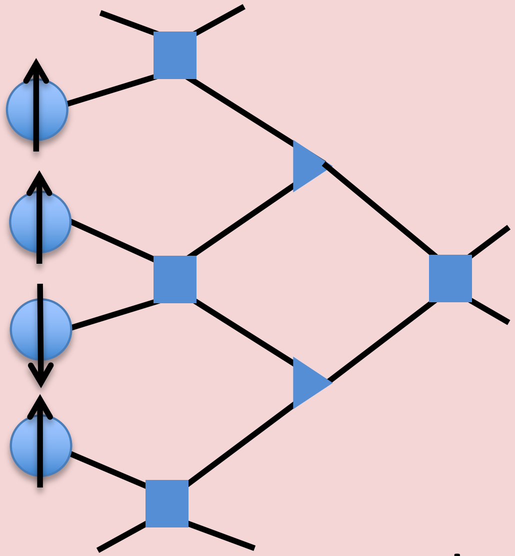


Neural network



Carleo, Troyer `17

|| Swingle `10



Tensor network

Roadmap

1.

Quantum gravity in (d+1)-dim.

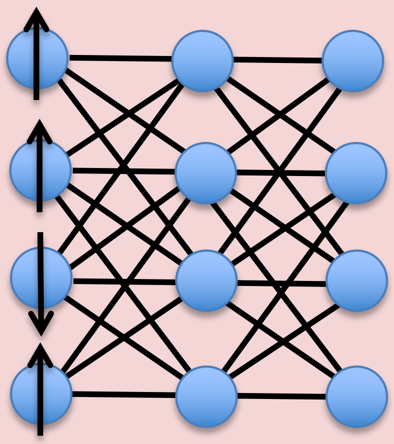
‘tHooft `93
Susskind `94
Maldacena `97

||

Quantum mechanics in d-dim.

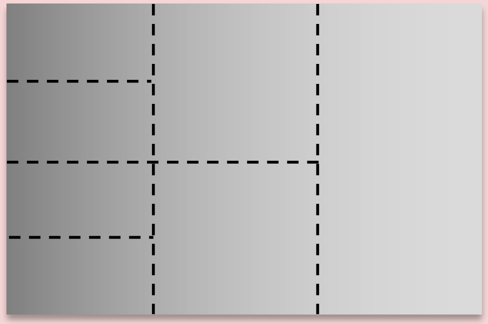
General spacetime

|| ?

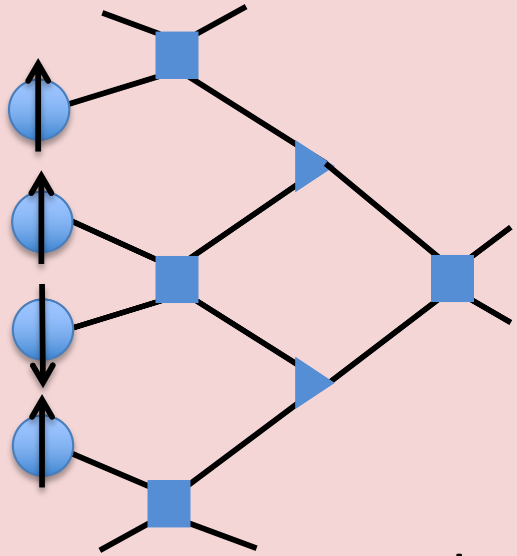


Neural network

Anti de Sitter spacetime



|| Swingle `10



Tensor network

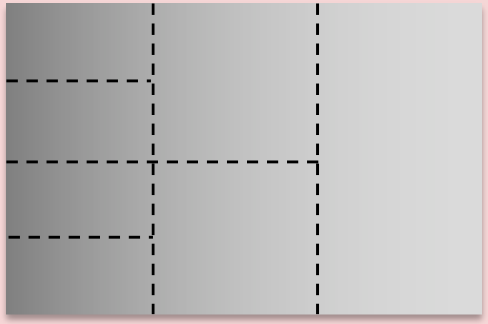
Carleo, Troyer `17

Roadmap

Quantum gravity in (d+1)-dim.

General spacetime

Anti de Sitter spacetime



‘tHooft `93
Susskind `94
Maldacena `97

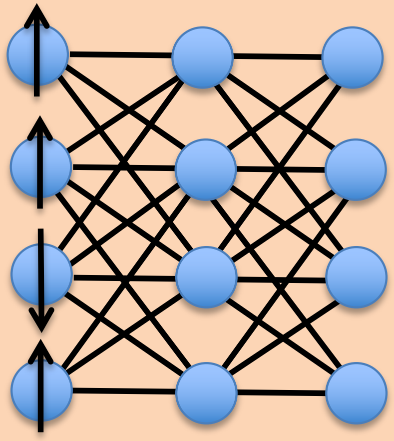
||

Quantum mechanics in d-dim.

|| ?

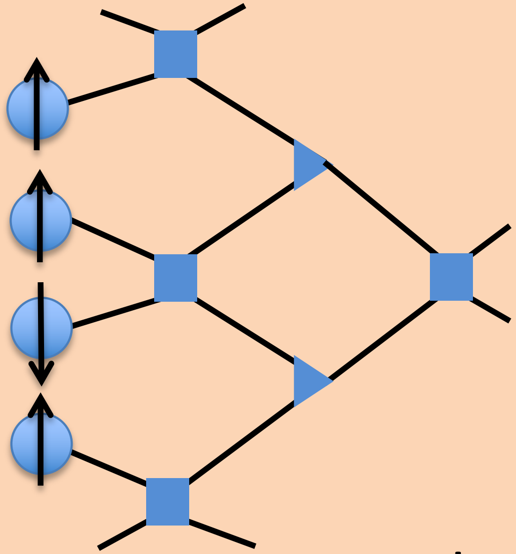
Swingle `10

2.



Neural network

Carleo, Troyer `17



Tensor network

Roadmap

Quantum gravity in (d+1)-dim.

‘tHooft `93
Susskind `94
Maldacena `97

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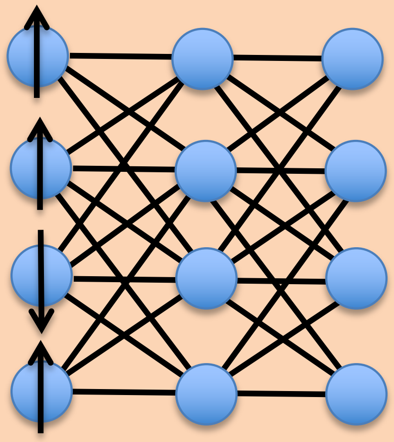
Quantum mechanics in d-dim.

3. General spacetime

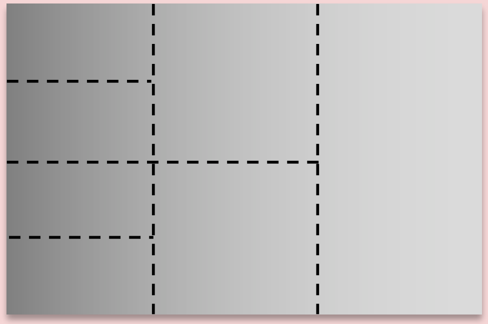
|| ?



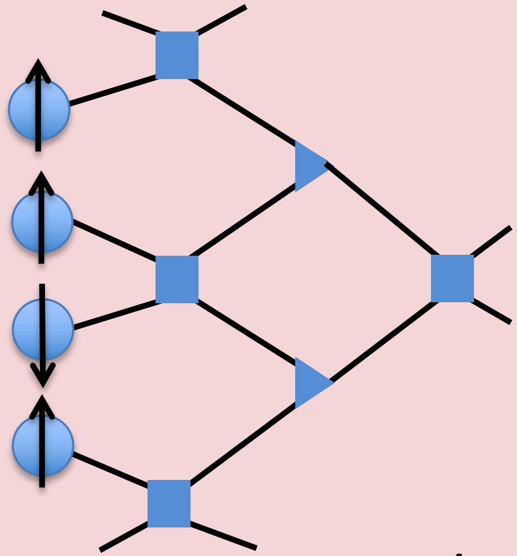
Neural network

Carleo, Troyer `17

Anti de Sitter spacetime



|| Swingle `10



Tensor network

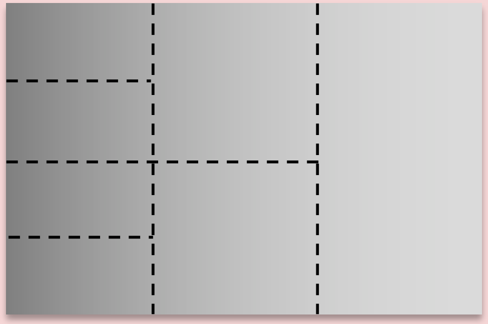
Roadmap

4.

General spacetime

Anti de Sitter spacetime

Quantum gravity in (d+1)-dim.



‘tHooft `93
Susskind `94
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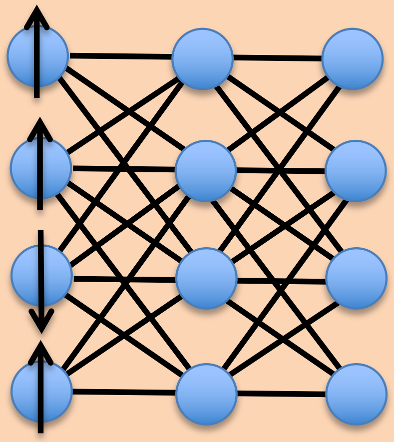
||

|| ?

|| Swingle `10

Quantum mechanics in d-dim.



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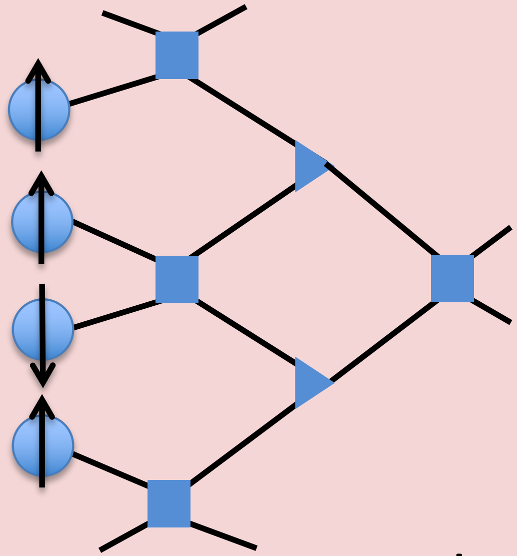


Neural network

Tensor network

Deep Learning and Quantum Gravity

1. Quantum gravity — 3 pages
2. Neural network quantum states — 3 pages
3. When is NN a spacetime? — 5 pages
4. Spacetime emergent from data — 4 pages

Discussion: Quantum gravity $\subset$ ML ?

① Quantum Gravity

Brief History of quantum gravity

1974 'tHooft, Veltman:
Perturbation fails in Einstein gravity.

1970 Nambu, Susskind, Nielsen:
String theory of hadrons.

1974 Yoneya, Scherk, Schwarz:
String is quantum gravity.

1971 Bekenstein:
Black hole entropy.

1993 'tHooft, Susskind:
Holographic principle.

1997 Maldacena:
AdS/CFT correspondence.

① Quantum Gravity

AdS/CFT correspondence, no proof

[Maldacena, Adv.Theor.Math.Phys. 2 (1998) 231]

"CFT"		"AdS"
"Large N" Quantum mechanics in d-dim. spacetime	=	Classical ~~Quantum~~ gravity in (d+1)-dim. spacetime

- No proof!
- Given Left, how can one get Right?
- Vast amount of evidence known
- Maldacena's paper is top-of-top-cite

① Quantum Gravity

Dictionary : equating partition functions

[Gubser, Klebanov, Polyakov, Phys.Lett.B428(1998)105]
[Witten, Adv.Theor.Math.Phys. 2 (1998) 253]

Partition function of Quantum mechanics = Partition function of Classical gravity

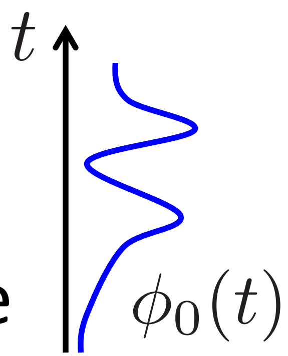
$$Z[\phi_0] = Z[\phi_0]$$

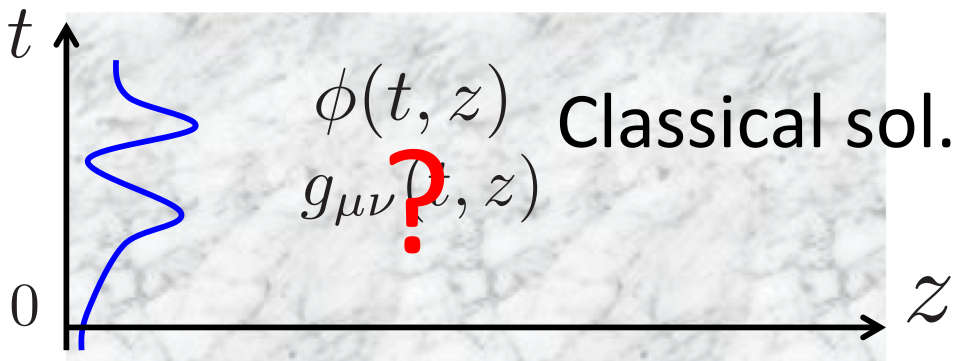
$$Z[\phi_0] = \int [\mathcal{D}q(t)] e^{-\int dt (\mathcal{L}[q,\dot{q}] + \phi_0(t)\mathcal{O}[q])}$$

$$Z[\phi_0] = e^{-\int dt dz \sqrt{-g}(R[g] + \mathcal{L}[\phi] + \cdots)}$$

?

source





$$\phi_0(t) = \phi(t, z=0)$$

Deep Learning and Quantum Gravity

1. Quantum gravity — 3 pages
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Discussion: Quantum gravity $\subset$ ML ?

② Neural Network Quantum States 1/3

Find ground state wave function $\psi(s_1, s_2, \cdots, s_N)$

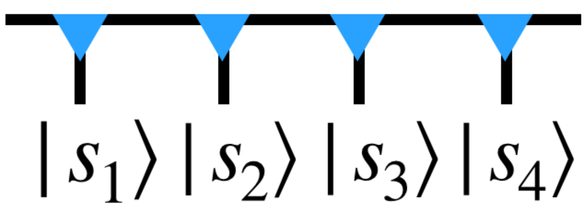
Q : Minimize its energy *E* for a given Hamiltonian *H*,

$$E = \frac{\sum_{s_1,\cdots s_N, s'_1,\cdots,s'_N} \psi^\dagger(s'_1,\cdots,s'_N)\hat{H}_{s'_1,\cdots s'_N,s_1,\cdots,s_N}\psi(s_1,\cdots,s_N)}{\sum_{s_1,\cdots s_N}\psi^\dagger(s_1,\cdots,s_N)\psi(s_1,\cdots,s_N)}$$

A : Use ansatz and optimize parameters!

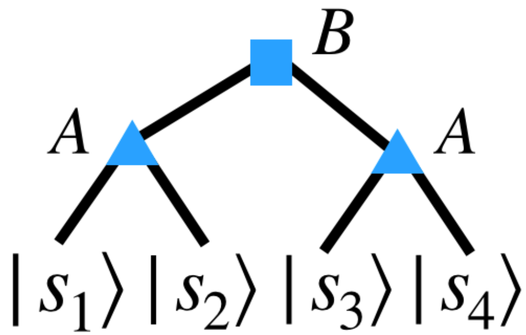
- Matrix product states

$$\psi(s_1, s_2, \cdots) = \mathrm{tr}[A^{(s_1)}A^{(s_2)}\cdots]$$



- Tensor network states

$$\psi(s_1, s_2, \cdots) = \sum_{m,n} B_{mn} A_{ms_1s_2} A_{ns_3s_4}$$

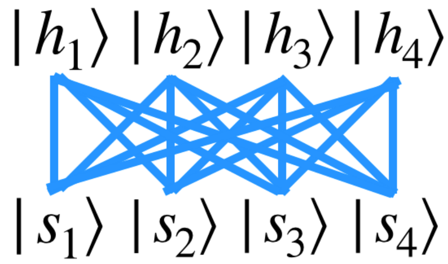


② Neural Network Quantum States 2/3

Neural network can be wave functions

- Boltzmann machine states

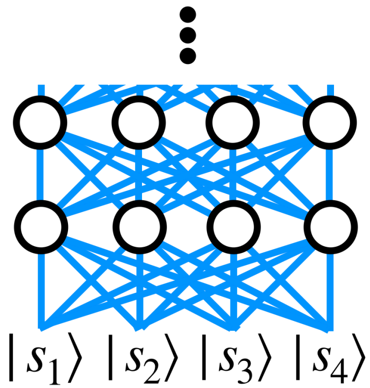
  [Carleo, Troyer `17],
  [Nomura, Darmawan, Yamaji, Imada `17], ..



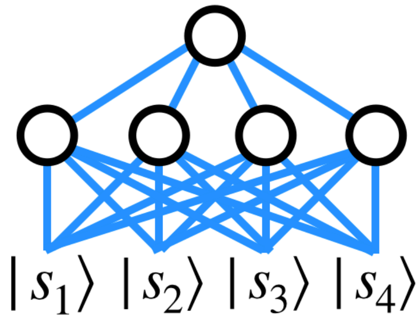
$$\psi(s_1, \cdots, s_N) = \sum_{h_A} \exp\left[\sum_a a_a s_a + \sum_A b_A h_A + \sum_{a,A} J_{aA} s_a h_A\right]$$

- Deep Boltzmann machine states

  [Carleo, Nomura, Imada `18], ..



- Feedfoward network states [Saito `18], ..



$$\psi(s_1, \cdots, s_N) = \sum_i f_i\, \sigma\left(\sum_j W_{ij} s_j + b_i\right)$$

② Neural Network Quantum States 3/3

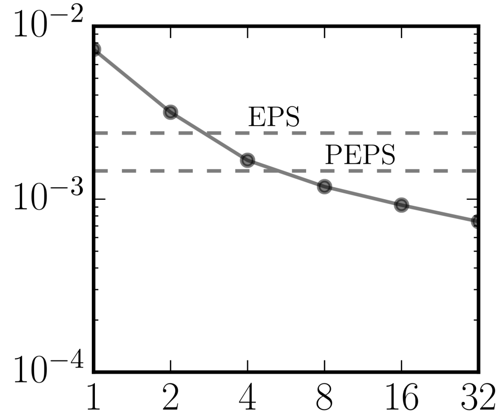
Better? and Why?

Neural states may beat conventional ones.

Ex) 2-dimensional antiferromagnetic Heisenberg model

[Carleo, Troyer `17]

Energy with RBM states



# of hidden units

Discovered intimate relations are there.

1) Boltzmann machine states are tensor network states

[Chen, Cheng, Xie, Wang, Xiang `18]

2) Tensor states are deep Boltzmann [Gao, Duan `17] [Huang, Moore `17]

3) Tensor states are feedforward with "product pooling"

[Cohen, Shashua `18]

Ex) Unified approach: MPO-Net [Gao, Cheng, He, Xie, Zhao, Lu, Xiang `19]

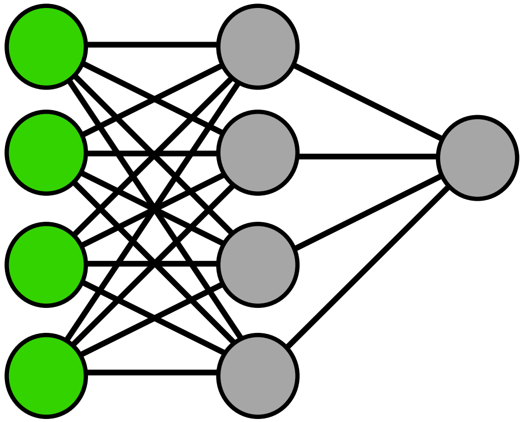
Deep Learning and Quantum Gravity

Discussion: Quantum gravity $\subset$ ML ?

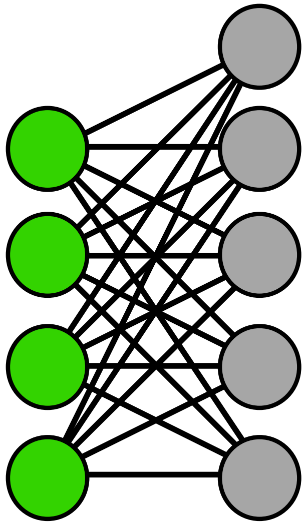
③ When is NN a spacetime?

General NN is not a space

No notion of which unit is close to which



Perceptron model
[Rosenblatt 1958]
[Rumelhart, McClelland 1986]

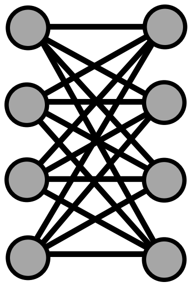


Boltzmann machine
[Ackley, Hinton, Sejnowski 1985]

③ When is NN a spacetime?

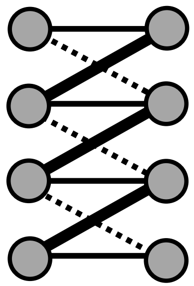
Sparsity + weight sharing, for NN to be a space

No locality



Fully connected

Locality imposed



Convolutional layer

[K. Fukushima `80]

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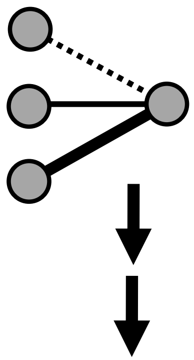


Parallelly translated

Input: $\phi(n\Delta x)$

Output:

$$a\phi(n\Delta x) + b\partial_x\phi(n\Delta x) + c\partial_x^2\phi(n\Delta x) + \cdots$$

③ When is NN a spacetime?

NN depth as time

Dynamical system

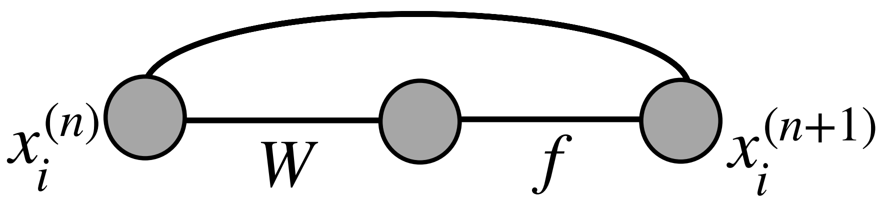
$$\dot{x}_i = f_i(x(t)) \quad \Longrightarrow \quad x_i(t_{n+1}) = \underline{x_i(t_n) +} \Delta t \cdot f_i(x(t_n))$$

$$t_{n+1} = t_n + \Delta t$$

Discretized time

ResNET (Residual network) : easily trained deep model

[K.He et al.,1512.03385]



$$x_i^{(n+1)} = f(W_{ij} x_j^{(n)}) \underline{+ x_i^{(n)}}$$

Skip connection

③ When is NN a spacetime?

Hamilton dynamics is a NN

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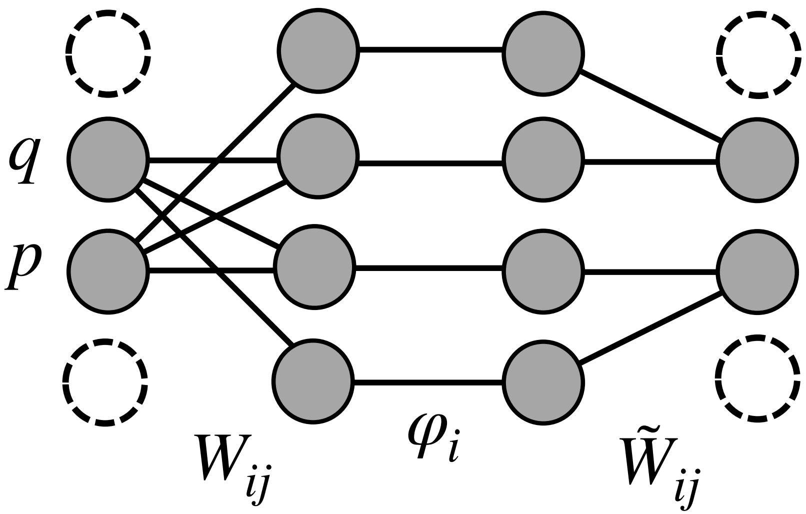
$$\dot{q} = \frac{\partial H}{\partial p}, \qquad \dot{p} = -\frac{\partial H}{\partial q}$$

Time-dependent Hamiltonian = weights/activation



$$W = \begin{pmatrix} 0 & 0 & v & 0 \\ 0 & 1+\Delta t w_{11} & \Delta t w_{12} & 0 \\ 0 & \Delta t w_{21} & 1+\Delta t w_{12} & 0 \\ 0 & u & 0 & 0 \end{pmatrix}$$

$$\varphi_i = \begin{pmatrix} \Delta t f(x) \\ 1 \\ 1 \\ \Delta t g(x) \end{pmatrix} \qquad \tilde{W} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \lambda_1 & 1 & 0 & 0 \\ 0 & 0 & 1 & \lambda_2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$H = w_{11}pq + \frac{1}{2}w_{12}p^2 - \frac{1}{2}w_{21}q^2 + \frac{\lambda_1}{v}F(vp) - \frac{\lambda_2}{u}G(uq)$$

$$(F' = f, \quad G' = g)$$

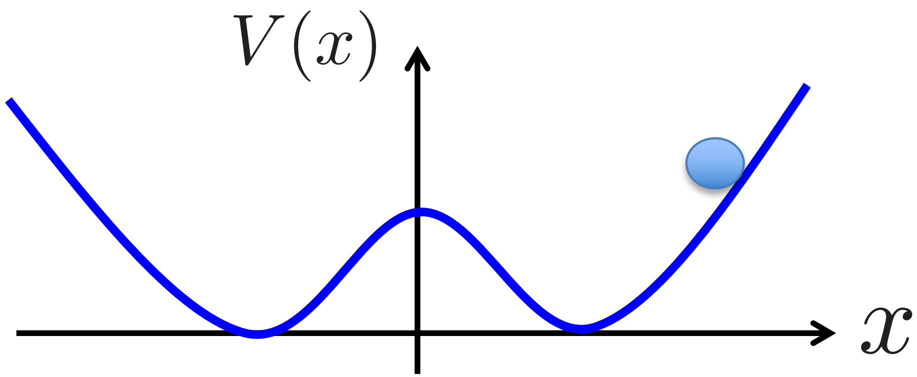
③ When is NN a spacetime?

Q. Find a Hamiltonian

Consider a particle motion $x(t)$ in a given potential $V(x)$ in 1 dimension, with unknown time-dependent friction force $h(t)\dot{x}$.
One tried many initial conditions $(x(t=0), \dot{x}(t=0))$ and collected those which stops at $t=10$.

Q. From given data of the initial conditions, find $h(t)$.

$$m\ddot{x} = h(t)\dot{x} + \frac{\partial V(x)}{\partial x}$$



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Discussion: Quantum gravity $\subset$ ML ?

④ Spacetime emergent from data

Gravity side

Classical scalar field theory in unknown 5-dim. spacetime

1802.08313
1809.10536

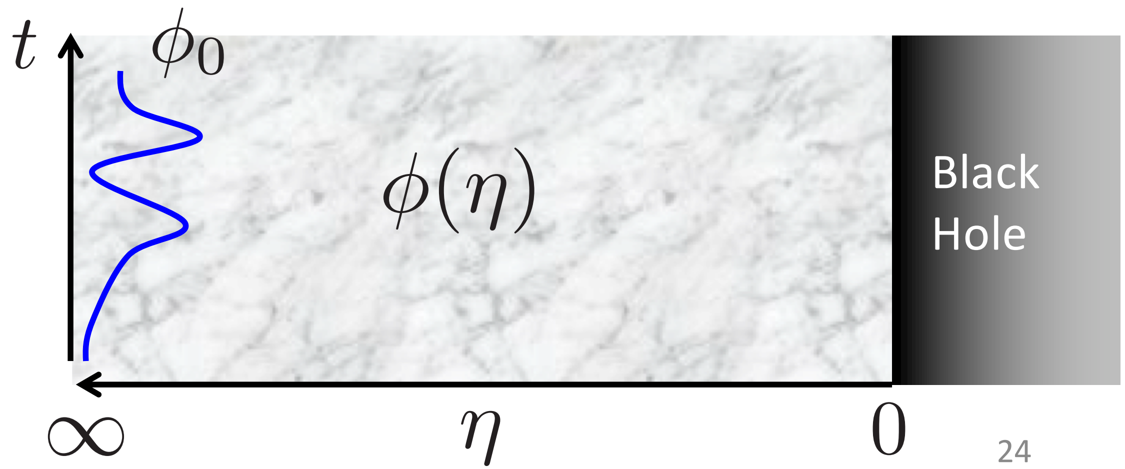
$$S = \int d\eta d^4x \sqrt{\det g}\left[(\partial_\eta \phi)^2 - V(\phi)\right]$$

$$\begin{cases} ds^2 = -f(\eta)dt^2 + d\eta^2 + g(\eta)(dx_1^2 + \cdots + dx_{d-1}^2) \\ V[\phi] = -\dfrac{3}{L^2}\phi^2 + \dfrac{\lambda}{4}\phi^4 \end{cases}$$

Data: $(\phi_0, Z[\phi_0])$

⇕

$(\phi|_{\eta=\infty}, \partial_\eta \phi|_{\eta=\infty}, \partial_\eta \phi|_{\eta=0})$



④ Spacetime emergent from data

Equation of motion as a feedforward NN

Eq. of motion $\partial_\eta^2 \phi + \underline{\underline{h(\eta)}} \partial_\eta \phi - \frac{\delta V[\phi]}{\delta \phi} = 0$

⇩

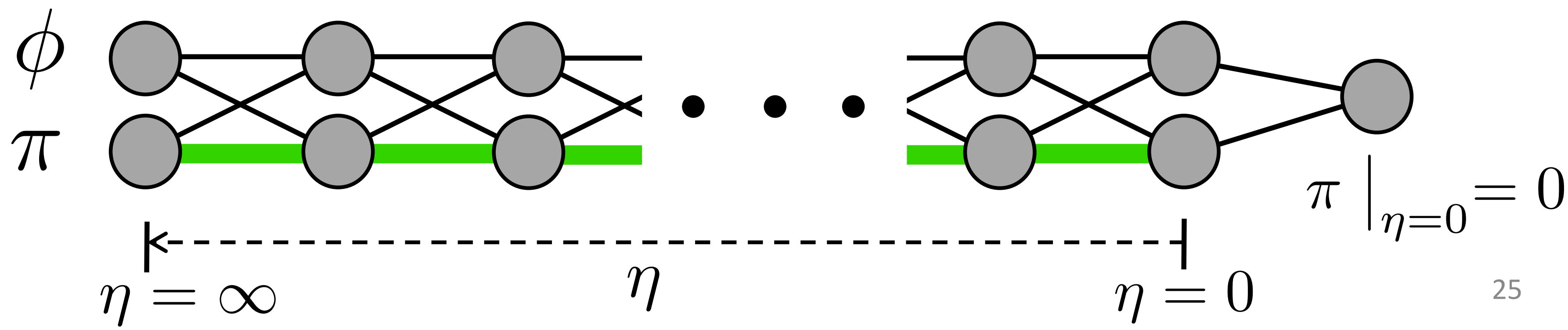
metric $h(\eta) \equiv \partial_\eta \left[\log \sqrt{f(\eta) g(\eta)^{d-1}} \right]$

Discretization
Hamilton form

$$\begin{cases} \phi(\eta + \Delta\eta) = \phi(\eta) + \Delta\eta\, \pi(\eta) \\ \pi(\eta + \Delta\eta) = \pi(\eta) + \Delta\eta \left(h(\eta) \pi(\eta) - \frac{\delta V(\phi(\eta))}{\delta \phi(\eta)} \right) \end{cases}$$

⇩

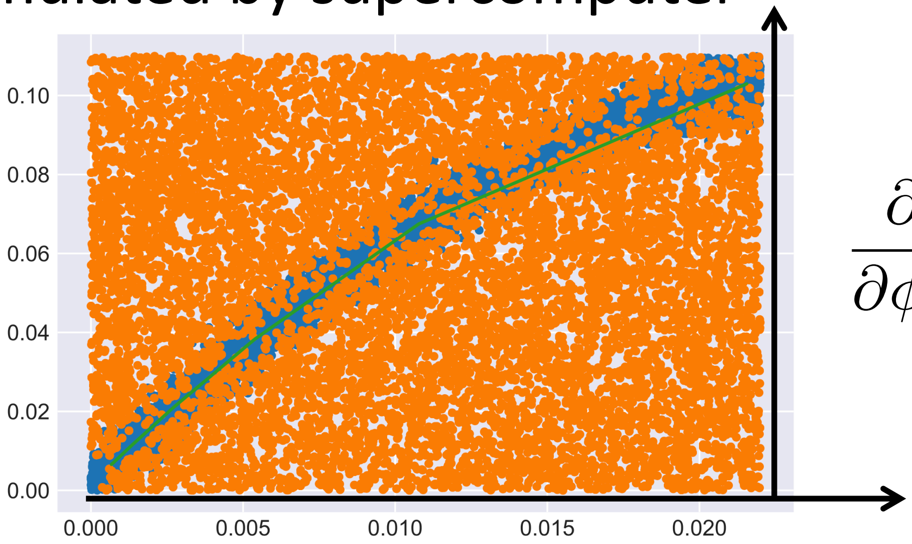
Feedfoward neural network for classification



④ Spacetime emergent from data

Training with data of quark condensate

Data of quantum chromodynamics
simulated by supercomputer



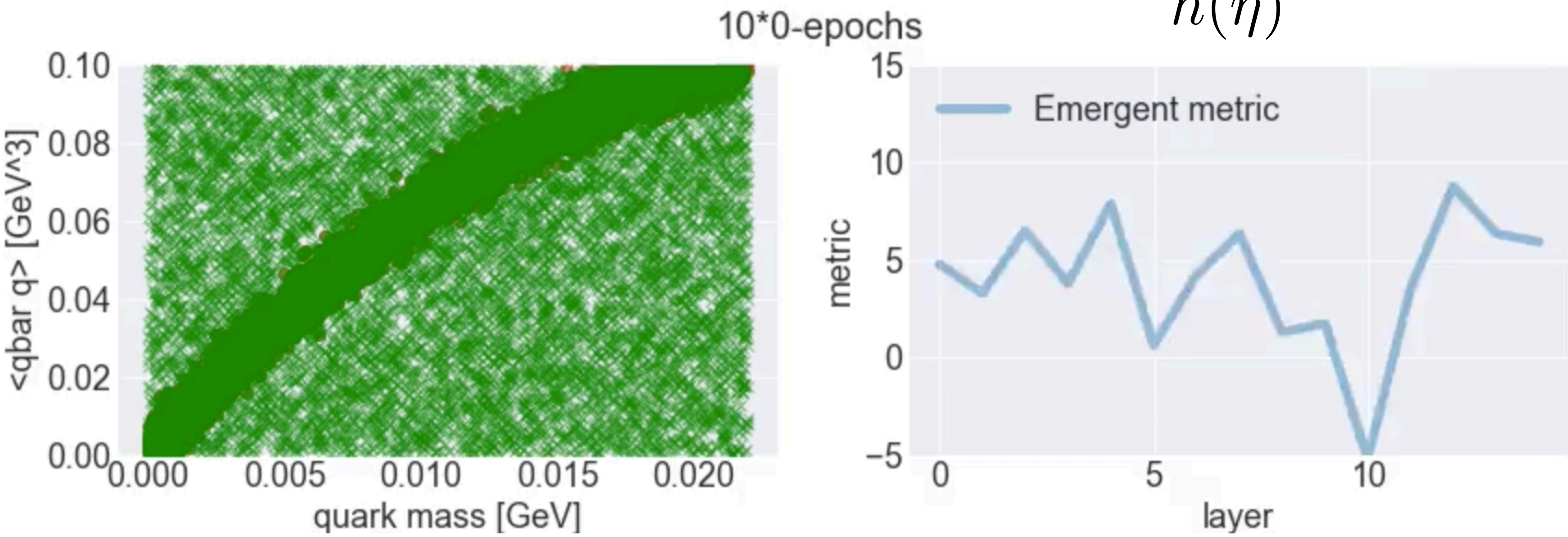
$\frac{\partial}{\partial \phi_0} Z[\phi_0]$: Quark condensate

ϕ_0 : Quark mass

④ Spacetime emergent from data

Training with data of quark condensate

$h(\eta)$



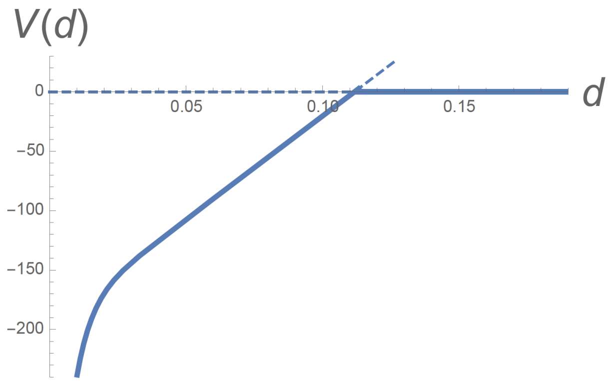
Trained values of potential :

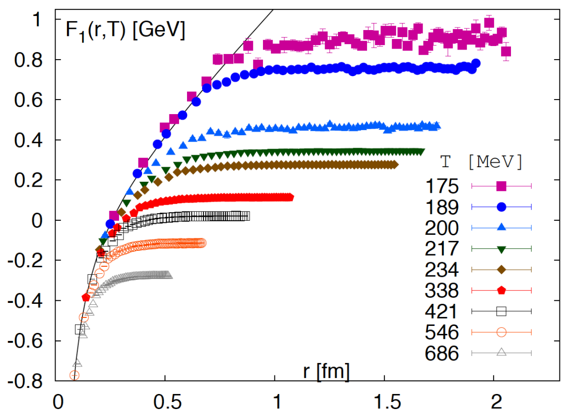
1/L = 237(3)[MeV] ,   λ/L = 0.0127(6)

④ Spacetime emergent from data

Emergent metric can predict physics

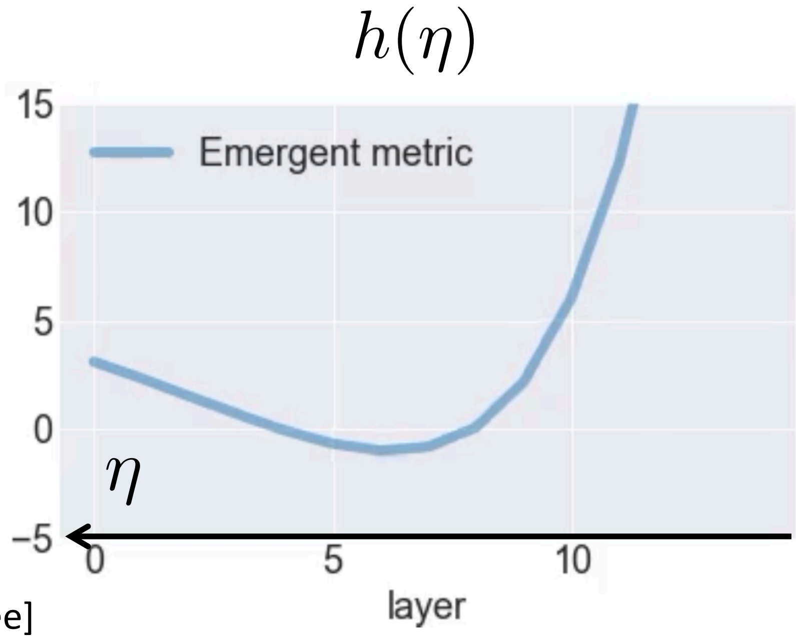
Interquark potential





[Petreczky, `10]

Procedures
based on
[Maldacena]
[Rey,Theisen,Yee]



Deep Learning and Quantum Gravity

Discussion: Quantum gravity $\subset$ ML ?

Discussion: Quantum gravity $\subset$ ML ?

3 steps for quantum gravity

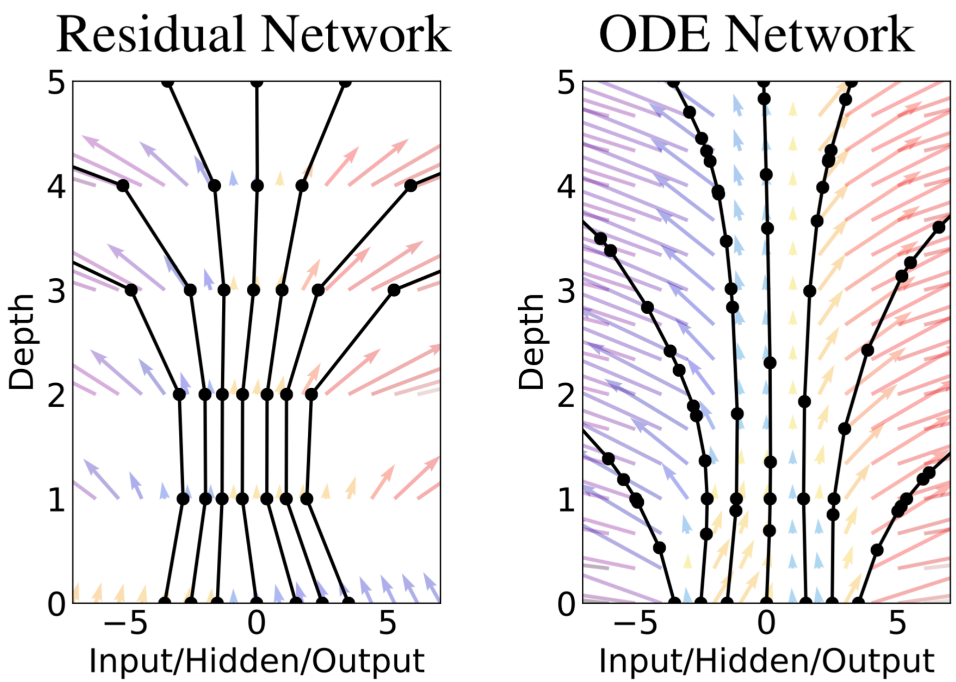
Quantum Mechanics side
Large DoF limit
Large DoF expansion
Finite DoF

Gravity side		Architecture
metric $g_{\mu\nu}$	field ϕ	
Classical	Classical	Feedforward NN
Classical	Quantum	Deep Boltzmann
Quantum	Quantum	?

- Neural ODE : free from discretization
- Quantum AdS/CFT $\subset$ Deep Boltzmann machine
- Conventional quantum gravity looks networks

Discussion: Quantum gravity ⊂ ML ?

Neural ODE : free from discretization



Neural ODE [R.T.Q.Chen, Y.Rubanova, J.Bettencourt, D.Duvenaud 1806.07366]

2006.00712

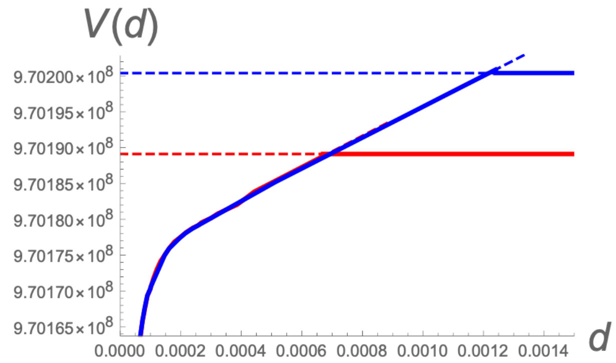
$$\frac{d\phi(\eta)}{d\eta} = f(\phi(\eta), \eta, h(\eta))$$

Emergent metric

$$\begin{aligned} h(\eta) = & 8.2351\tilde{\eta}^8 + 8.0108\tilde{\eta}^7 + 7.6071\tilde{\eta}^6 \\ & + 6.9468\tilde{\eta}^5 + 150.8853\tilde{\eta}^4 - 130.8117\tilde{\eta}^3 \\ & + 55.5384\tilde{\eta}^2 - 2.22235\tilde{\eta}^1 + 3.7719. \end{aligned}$$

$$\tilde{\eta} = 1 - \eta$$

Q Qbar potential

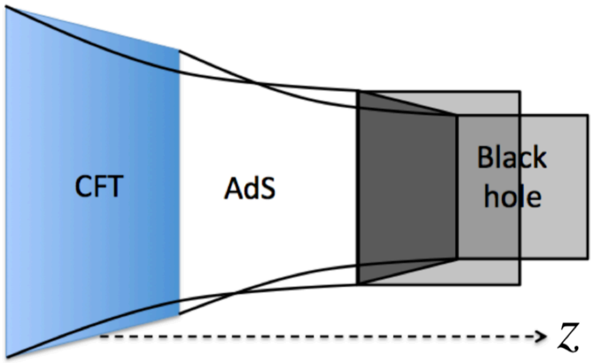


Discussion: Quantum gravity ⊂ ML ?

Quantum AdS/CFT ⊂ Deep Boltzmann

AdS/CFT

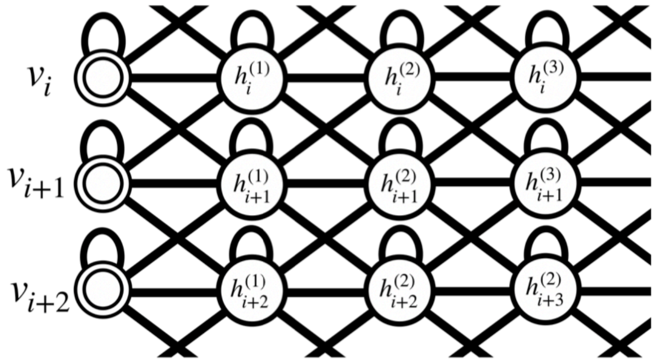
[Maldacena 1997]



$$Z_{\mathrm{QFT}}[J] = \int_{\phi(z=0)=J} \mathcal{D}\phi \, \exp\left(-S_{\mathrm{gravity}}[\phi]\right)$$

Deep Boltzman machine

[Salakhutdinov, Hinton 2009]



$$P(v_i) = \sum_{h_i \in \{0,1\}} \exp\left[-\mathcal{E}(v_i, h_i)\right]$$

[KH `19] [You,Yang,Qi `18] (See also [Gan,Shu `17][Howard `18])

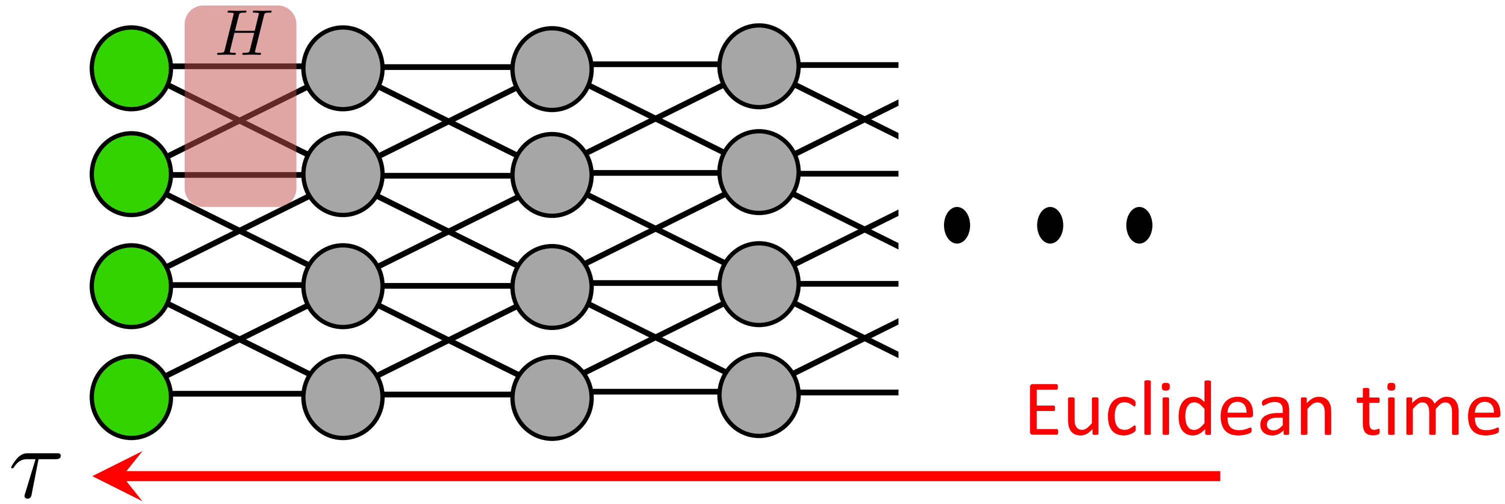
Discussion: Quantum gravity ⊂ ML ?

Physical picture of Deep Boltzmann

Ground state wave function for given Hamiltonian is identified as a deep Boltzmann machine

[Carleo, Nomura, Imada `18], ..

$$|\psi\rangle = \lim_{\tau\to\infty} e^{-\tau H}|\,\mathrm{any}\rangle = e^{-\Delta\tau H}e^{-\Delta\tau H}\cdots|\,\mathrm{any}\rangle$$



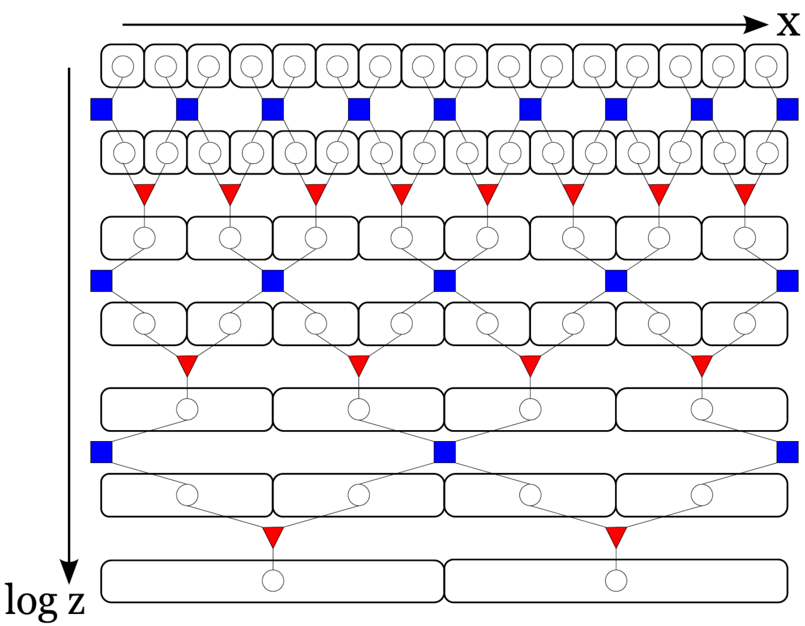
$$\psi(x_i) = \sum_{h_j^{(n)}\in\{0,1\}} \exp\left[-\sum_{ij} w_{ij}^{(0)} x_i h_j - \sum_n \sum_{ij} w_{ij}^{(n)} h_i^{(n)} h_j^{(n+1)}\right]$$

Discussion: Quantum gravity ⊂ ML ?

AdS/CFT discretized the bulk, but fixed

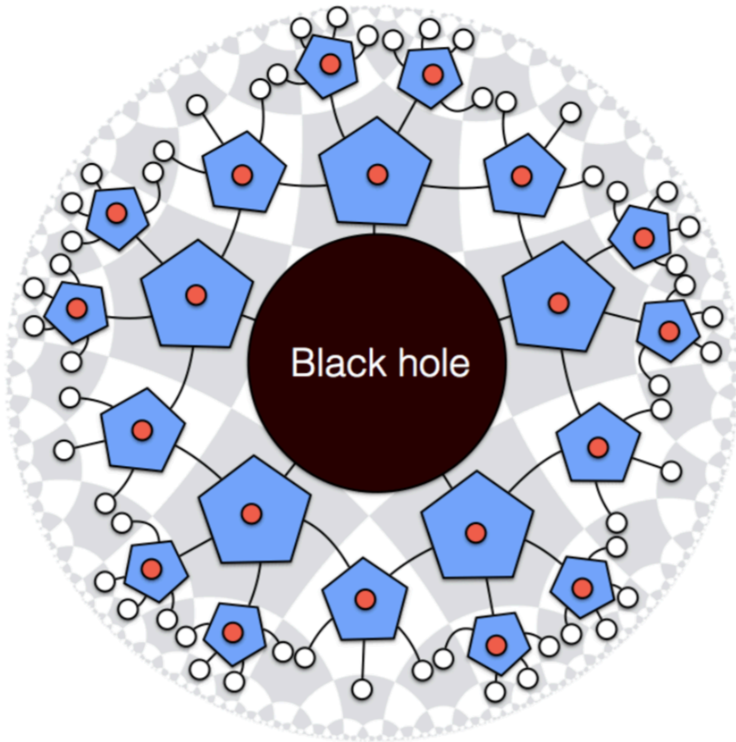
AdS/MERA

[Swingle `09]



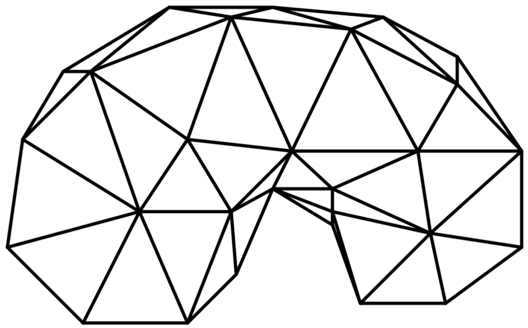
Quantum codes for holography

[Pastawski, Yoshida, Harlow, Preskill `15]



Discussion: Quantum gravity ⊂ ML ?

Quantum spacetime? Regge vs Matrix



Regge calculus
[Regge `61]

Fixed lattice architecture,
variable lengths

⇩

Suits conventional NN

Dynamical triangulation
[Ambjorn, Loll `98]

Randomly generated
lattice architecture,
fixed lengths

⇩

Novel “QG NN”

Roadmap

Quantum
gravity
in (d+1)-dim.

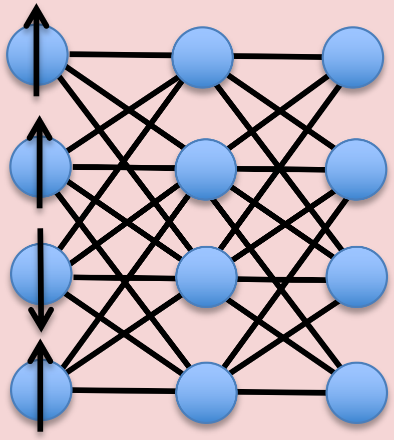
‘tHooft `93
Susskind `94
Maldacena `97

||

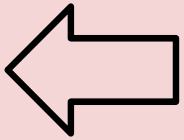
Quantum
mechanics
in d-dim.

General
spacetime

|| ?

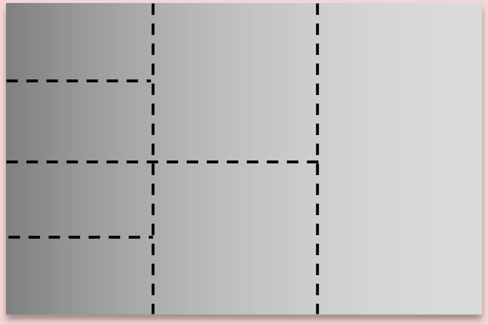


Neural network

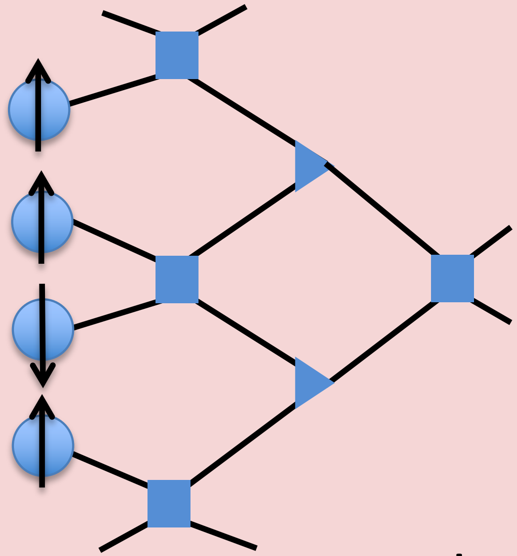


Carleo,
Troyer `17

Anti de Sitter
spacetime



|| Swingle `10



Tensor network